

Werner Heisenberg's Ideas on Particle Physics in the Light of Recent Achievements

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Dedicated to Hans-Peter Dürr on the occasion of his 60th birthday

W. Heisenberg's unconventional – at his time even revolutionary – ideas on “elementary” particles are reviewed – or, better, rediscovered – within the framework of present day gauge- and string-theories.

I. Looking back at a period long past, i.e. more than four decades ago, unforgettable hours of private discussions with Werner Heisenberg come back to my mind: I now realize that his then perfectly new physical ideas which, at that time, were not really appreciated, acquire by now, i.e. if re-interpreted in the light of recent and breathtaking developments, a perfectly new and unexpected physical meaning. In other words: Heisenberg's original principles [1] – if reformulated within the framework of present day new and far reaching mathematical concepts and experimental achievements – represent, in fact, an intuitive insight into a future theory long before time was ready for its implementation. It thus seems to me that the sixtieth birthday of Hans-Peter Dürr, one of Heisenberg's most devoted collaborators and successors, is the right moment to recall – in particular for a younger generation – the story of that fascinating period of scientific doubts and struggles leading to the creation of particle physics of our time, e.g. gauge- and string-theories, which appears in a way to be guided – as we will see in these short and incomplete lines – by Heisenberg's intuitions, or even visions, if I might be allowed to say so.

II. Let me just start with a lecture of Heisenberg which I occasionally heard as early as 1946 just after the important, even decisive discovery (by Occhialini) of the pion, i.e. the particle theoretically foreseen by Yukawa. It appeared – at that time – to be a real revelation, and the solution of an apparently clear-cut contradiction: The famous experiment at “Piano Cervino” (the laboratories of that time were the high

atmosphere and the outer space) by Pacini, Piccioni and Conversi (difficult to pronounce – the third name was usually replaced by “Conversly” – and even more difficult to understand) had clearly shown that the then well-known cosmic ray particles (later on called muons), whose mass and creation cross sections were apparently in accordance with Yukawa's famous prediction, did, however, not exhibit the expected relatively large absorption cross section as foreseen through a most elementary quantum-mechanical argument.

After a few months of questioning and discussions, Occhialini's wonderful combined photographic plates (exposed to the Bristol cyclotron) definitely showed that the particles produced in the highest layer of the atmosphere by cosmic protons (very much in the same way as in a cyclotron) exhibit, in fact, a spontaneous decay in flight long before they interact with matter on the earth, i.e. the photographic plates at “Piano Cervino”. The decay of these new particle, i.e. the true Yukawa-particles (by now called pions) into the particles observed in cosmic rays (i.e. the muons) with slightly lower mass and (observed) small interaction cross section, immediately solved all contradictions. Instead of expounding on the (at that time lengthy discussed and, in fact, evident) solution of this problem, Heisenberg took a quite unexpected and, for that time, perfectly new attitude: He nicely drew a vertical line on the blackboard indicating with dots the various mass values of the then known particles (i.e. electron, muon, proton, etc.) and thereupon frankly declared:

“Look here, there is, by now, another dot, i.e. another mass value (that of the pion) or, much better, another spectral line of a (most probably) infinite mass-spectrum, which has to be – and this

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was the decisive point – re-interpreted as a spectrum of energy eigenvalues of a still unknown ‘Hamiltonian’ or, better, unknown quantum-mechanical system.”

This fundamental assumption [1] exhibits, first of all, according to Einstein’s well-known principle, the unification of the two different – but equivalent – forms of energy, namely mass- and field-energies. As seen from a more physical, i.e. empirical viewpoint, this principle immediately explains naturally the apparent irregularly or, better, non-triviality of this mass-spectrum in perfect analogy to all well-known examples of eigenvalue-spectra. The same holds for the characteristic assignments of spin, isospin, etc. which are, in principle, perfectly analogous in the two cases, i.e. particles and eigenstates. On the other side, this new – by now perhaps evident – viewpoint leads to the elimination (at least to a large extent) of the “beloved” concept of elementarity in particle physics which, however, was still to survive – especially in the perfectly unjustified case of hadrons – for many years.

III. The major aim of this (by far too short) contribution is, in fact, to show that this principle, stated by Heisenberg at a period too early to be fully implemented, mainly because of lack of a few decisive empirical facts, represents in retrospect the guideline of all later research up to our days and might, therefore, righteously be called a scientific vision. On the other side, its explicit realization through later research looked rather different, i.e. by far more complex or, better, by far more spectacular (and, in a way, not to be foreseen nor expected) than Heisenberg originally might have assumed [2].

In fact, present-day non-abelian gauge theories [3], in particular QCD, fulfill – as we will see – Heisenberg’s principle in an enlarged sense, and I would like to maintain that even latest theoretical attempts towards an all englobing basic theory, i.e. the so-called strings [4], should be seen under this same aspect. If we include, for the time being, this – rather problematic – second step, it may be realized that Heisenberg’s quest for a complete elimination of the conventional particle concept (to be replaced by “geometry”, i.e. an enormously enlarged Eigenvalue problem!) appears to be satisfied (cp. the Appendix).

IV. Before engaging, however, into these far-reaching and basic questions, let us try to reconstruct the arguments (within the scientific framework of that

early period) which might have led Heisenberg to these general ideas which intuitively foreshadowed the future: Having worked in almost all important domains (as easily seen from the enormous list of his publications), he might have realized that practically all physical domains of research (molecules, atoms, nuclei, etc.) appear to be characterized by a system of spectra, however, spectra of very different magnitudes and different physical significances. On the other hand, they definitely exhibit very similar general properties (e.g. their general characterisation through the underlying symmetry group with the corresponding representations) and are, from the physical side, based – in each case – on a characteristic and more or less basic interaction or, better, interacting field, between the elementary constituents (e.g. phonon exchange between the (conduction-) electrons in normal superconductors; chemical forces (i.e. exchange-terms) in molecular band spectra; the electromagnetic force (i.e. photon exchange) in the case of atomic spectra and, finally, the so-called nuclear forces (e.g. meson-exchange) in nuclear spectra.

It thus seemed natural that a perfectly new and basic (so far unknown) field was definitely needed for generating this new particle spectrum, a quest appearing, however, strange to most physicists of that time.

In view of our analogy one should have introduced, in fact, two fields describing a new kind of basic particles interacting in turn through another new field. Heisenberg, oversimplifying things, introduced, or, better, “invented” one single spinorial field endowed, however, with a self-interaction, i.e. in field theoretical terms, a field satisfying a non-linear field equation [2]. As far as the special choice of this field equation was concerned, he invoked general invariance principles as well as simplicity. In order to describe special massless particles, he suggested (this, again, was new and unconventional!) the possibility of degenerate vacuum states (by now this idea is well established; but it was terribly criticized at his time) and the corresponding massless Goldstone particles.

V. Without going (for this moment) into further details, e.g. the unfortunate German expression “Urfeld” (for the new field) with its philosophical touch and a definitely too simplifying single (but non-linear) field equation, let us jump over two, maybe three decades of misunderstandings (with even tragic consequences) and state at once the present-day achievements: As a first step, we thus describe the

theory of hadrons, by now based on QCD: It exemplifies in a most striking form Heisenberg's general principles mentioned above:

1. The hadrons (i.e. baryons and mesons including [5] even nuclei(!)) do, in fact, form a infinite system of quantum states consisting on the (non-linear(!)) gluon gauge-field, which, in turn, provides the binding of the new basic constituents, i.e. the quarks: The enormous realm of all hadronic states is thus interpreted by a consistent theory (QCD in this case) characterised and fully determined by a general symmetry principle, i.e. the local invariance or gauge principle [3] based on the group $SU(3)$. In this way even Heisenberg's quest for basic symmetries appears to be realized in a most stringent form, however with respect to a Lie-group which was, at that time (the decisive concept of 3-valued colour was lacking!) by no means to be expected. It should be emphasized that the general structure of the conventional physical eigenvalue problems, as mentioned above, appears, in fact, again realized: It thus looks worthwhile to make a direct comparison between the two most important physical examples (based, in turn, on quantum chromodynamics (QCD) and quantum electrodynamics (QED)) with respect to their basic constituents, their typical Lie-groups (defining the decisive local gauge-invariance) and their domains of physical application, which, in both cases, exhibit an enormous realm of empirical data:

Electrons	$U(1)$	QED	Atoms	Molecules
↓	↓	↓	↓	↓
Quarks	$SU(3)$	QCD	Hadrons	Nuclei

Although the second line (i.e. QCD) is hampered by serious mathematical difficulties appearing in all applications (a.o. the non-Abelian character of the gauge-group $SU(3)$ generates automatically the non-linearity of the corresponding gauge-field equation), there is, in fact, not the slightest doubt about its physical validity: Empirical facts ranging from high energy "jets" to nuclear level schemes as well as the inner structure of hadrons with the corresponding mass-spectrum become, qualitatively, understandable.

A far more detailed treatment represents, however, the great challenge for the present generation: While writing these lines it turned out that various younger scientists [6] had just achieved a real "breakthrough" within this framework, in particular with respect to the decisive confinement problem. In this connection it might be remarked that this decisive property,

which appears to be a typical consequence of the non-linearity, again corresponds, in a way, to one of Heisenberg's general hypotheses, which states that these new basic fields might no longer be interpreted as (conventional) particle fields. Very much the same holds for his prediction about the degeneracy of the vacuum and the existence of the corresponding Goldstone particles: In fact, the pion, to be interpreted by now within the framework of QCD, just represents (at least partly) an important physical example:

It appears, in a way, an interesting, even striking fact, that Heisenberg used just this case for exemplifying his ideas: Its typical Goldstone behaviour (as established through QCD) yields the most characteristic example of non-elementarity. It is, in this connection, rather amusing to remember that the pion-field was considered – even for a longer period – to be the elementary boson field (to be compared with the Maxwellian field in atoms) yielding nuclear binding! The real analogy to the photon (Maxwell) field is, of course (as indicated above), the gluon (gauge) field which replaces the former "boson-exchange" and represents – in view of the unsurmountable difficulties (and even contradictions) of the former viewpoint [5] – a real revelation and the starting point of a perfectly new (but more basic) nuclear theory!

2. This "fulfillment" of Heisenberg's principles covers, however, only the hadronic (nuclear) part of physics. The remaining basic particles (i.e. leptons and quarks) are, so far, still to be called elementary! As a second point of this contribution I would like to state: Within the framework of modern string theoretical attempts these remaining particles will again be re-interpreted as quantum states of another but perfectly "new" and unconventional elementary system: The strings are two-dimensional "tubes" enlarging in a way the conventional particle lines in Feynman's diagrams, immersed, however, into a high-dimensional space, which, in turn, generalizes the five-dimensional space, introduced already at a very early stage by Kaluza and Klein. Although this second step is far from being fully worked out and appears not generally acceptable, it is interesting to observe that Heisenberg's basic viewpoint (particles as eigenstates) is again realized (however, in an enormously enlarged and abstract setting!) through a (generalized) quantisation scheme of a "classical" two-dimensional field.

The short but "euphoric" period of string-theories (interpreted occasionally as the "theory of every-

thing”), which soon faded away, led, however, in view of its deep-lying geometrical (i.e. topological) ingredients to an enormous and far-reaching development of mathematical physics which, if I might say so, lead nearly to a unification of theoretical physics with research in modern (abstract) mathematics. Examples are the impressive relations between statistical mechanics and conformal field theory, the renewal of and unexpected enlargement of “knot-theory”, etc. One might thus remark that Heisenberg – definitely without having this intention – led us to embark in a new direction of research: In fact we discover new and unexpected relations between empirical physical facts and abstract mathematical thoughts as emphasized a.o. also by leading mathematicians [8].

VI. After this most abbreviated overview of four decades of theoretical research I should give some more details (and adding a few rather personal remarks) on this critical lapse of time between the first ideas of 1946 and their (at least partial) realization in the sixties: First of all, Heisenberg’s own attempt of a practical realization of his general principles, i.e. his “famous” non-linear spinorial field equations [2] led to severe critics and great misunderstandings: Unfortunately, this simplifying (and, in fact, tentatively introduced) field equation was – mainly through repetitions in newspapers all over the world – advertised and even hailed to be the allenglobing basic theory, i.e. the “Weltformel” (world formula) more or less in analogy, if not as a realization, of Einstein’s well-known “vision”.

Coming again down to earth, it may, however, by now be realized that this world formula appears practically to be identical with one of the well-known bag-models, i.e. an extremely simplifying and approximate treatment of QCD applied to the internal structure of the nucleons, thus indicating that Heisenberg had, so to speak, guessed intuitively an approximation scheme for QCD, i.e. a theory definitely not to be fully developed at his time (the decisive half empirical concept of colour, i.e. the basic input for QCD, was not yet available).

The real tragedy – hinted to before – began, however, with the mathematical problem of solving the “world equation”: Remembering his close and incredibly fruitful collaboration with Wolfgang Pauli already in the case of his first formulation of quantum mechanics (cp. the edition of Pauli’s letters where he suggests two ways of looking at the atomic spectra: i.e. through the “space-optics” or through the “momen-

tum-optics”) and, more important, in the foundation of relativistic quantum field theory (QFT, i.e. the basis of physics for a long period and partly until now), Heisenberg asked for a renewed collaboration and, in particular, for help in mathematics. Pauli, after strong hesitations and even “sneering” remarks, all of a sudden agreed to this offer and (what was even more striking) he did so quite enthusiastically. His reason, i.e. his scientific argument for this sudden change had, in fact, to do with that most tragic outcome of this renewed collaboration from the very beginning: Pauli saw in Heisenberg’s idea a new way and a great hope for solving his own problem, i.e. his deep dissatisfaction and critics of the mathematical consistency of QFT, created, as mentioned above, in collaboration with Heisenberg. Thus, there were two different (even opposite) aims pursued by two great scientists: On one side (according to Heisenberg) the new theory should – through its intuitively introduced structure – reproduce reasonably well a large domain of empirical facts (here masses and internal properties of particles); on the other side (i.e. Pauli’s quest in all parts of theoretical physics) the “new” structure should obey the strict rules of mathematical consistency! Heisenberg occasionally remarked that these two, so to speak extreme or, better, opposite requests in physical theories could never be solved by one scientist alone. Nevertheless, Pauli – undoubtedly feeling, but not really realizing that not much time was left for him – started to work with unprecedented enthusiasm and hope without taking slightest care for his health. Heisenberg – in a way seduced by Pauli’s absolutely unusual optimism – might have taken things too easy and may, perhaps, not have opposed strongly enough the really exaggerated “fuss” made by newspapers on his “Weltformel”.

The breakdown (in fact to be expected) after two months of “day and night” work, i.e. the insight of enormous difficulties (a.o. to reconcile the two opposite ends) came all too sudden and even in a most dramatic form: Pauli, acting like (or, better, being) a “pope”, put the very first preprint on the “index” (of course with the effect that no other paper ever found such a widespread interest) and (what was by far worse) accused Heisenberg openly of talking nonsense, even in the course of a highly official conference. To my opinion it was perhaps Pauli’s – maybe psychologically understandable – reaction on his own failure, or, better, on a scientific defeat he had suffered (in a state of nervous depression) in a seminar given

earlier at Columbia University as well as his “hate” as regards the absolutely “out of place” publicity of the “Weltformel”.

VII. Accidentally, I was, with deep sorrow, reminded to this tragic end of the formerly unbelievably important collaboration between two of our greatest scientists: Many years after Heisenberg's death I heard (from a close relative of his) how much he had suffered – without having shown this in the slightest way in public – as a consequence of Pauli's irresponsible attacks which, in turn, encouraged most inappropriate scientific criticisms from all kinds of incompetent people during all his later years. Pauli's, maybe, last words to Heisenberg: “Werner, you please go on (with your research), I am no longer able to do so” were, apparently, not generally known.

It appears in a way tragic that Heisenberg could not – maybe caused by these unpleasant conditions – realize (and enjoy) that the then starting new developments were due, to a large extent, to his own ideas: In fact, he had introduced, already at a much earlier stage, the fundamental concept of an inner degree of freedom (i.e. in his case the “isospin”) decisive for all non-abelian gauge theories. This basic concept was, in fact, generalized by Gell-Mann (through his wonderful group-theoretical analysis of the empirical Hadron mass-spectrum, assumed – again according to Heisenberg – to be a physical level-scheme of a so far unknown system) by the introduction of the “colour” degree of freedom. It seems to me that Heisenberg lacked – maybe for the reasons mentioned above – the needed contacts to theoretical and experimental research of this time in order to be convinced about the existence of this triple-valued colour which, in turn, represents the (empirical) input for formulating this basic local $SU(3)$ gauge invariance leading to QCD, i.e. the underlying (mathematically consistent and all-embracing) theory of this enormously extended realm of modern hadron physics (cp. the Appendix).

Although QCD with its 8-component (gluon-) gauge field and the “coloured” fermions (quarks) is by far more complicated than Heisenberg's non-linear spinor field, it should be stressed that the first one appears entirely determined (up to the scale-constant) through the basic request of local gauge-invariance which, in turn, exhibits a formal analogy to General Relativity and appears to be by far the most natural (even, in a way the simplest) assumption leading to an understanding of elementary empirical facts (e.g. the

colourfree bound states, the observed octets and decuplets, a.s.o.). Local gauge invariance, i.e. an extreme example of Heisenberg's request for symmetry, guarantees at the same time renormalizability, i.e. properties apparently not shared by his model which, therefore, was hampered by unsurmountable mathematical difficulties.

VIII. Coming back to Pauli, I realized (at the occasion of an unforgettable and long walk in common through a deep forest on some Jura mountain only a few weeks before his untimely death) that he was deeply depressed for not having reached his goal of a logically consistent theoretical basis of physics. His message, however, which leads us directly into present-day research, reads: A great effort and a decisive scientific step – of the same size as the one taken in connection with the foundation of quantum mechanics in the twenties – will be needed in order to reach a deeper and more satisfactory understanding of nature. In that connection he remarked several times that the difficulties were apparently related to the use of an Archimedian, i.e. “normal” structure of space even in smallest dimensions: “All difficulties are caused by the concept of the ideal space-time point”. These words appear, by now, to be a real vision which foreshadows the recent efforts to find a substitute for the usual space-time structure within the framework of “p-adic” spaces, or, better and by far deeper, the introduction of the non-commutative geometry of Connes and Kastler [7] (cp. the Appendix).

These efforts point, in a way, to a final reconciliation of the two opposite viewpoints discussed above: If one visualizes – as hinted to before – the string as a remote outcome of Heisenberg's ideas about the geometric interpretation of mass spectra, it could be emphasized that the string concept with its extreme dimensions (i.e. the Planck length) leads, in fact, theoreticians to reconsider and change the usual concept of basic space-time in the way mentioned here and foreseen (as sometimes forgotten!) already by Riemann in his famous thesis on his geometry.

May I thus end this short and, maybe, superficial survey of an interesting and “critical” part of recent developments in physics by stating that Heisenberg's ideas – I insist to use the term visions – are (after such a long period of misunderstandings) again with us: They foreshadowed, so to speak, present-day far-reaching “geometrization” and the enormous enlargement of basic concepts. Within the framework of these

string-theoretical attempts the unconcilable quests of Pauli and Heisenberg (with their tragic outcome) are, in a way, reconciled: These new attempts correspond on one side to a (remote) application of Heisenberg's principles, on the other they practically enforce – through their extreme dimensions – a change of space-time structure as hoped and even foreseen by Pauli. In this way, the basic ideas of the two unforgottable scientists are reconciled on this much deeper (but still remote) level of future research.

Appendix

In these short lines the present day “realisation” (i.e. modern Gauge-theory) of Heisenberg's ideas mentioned above will be characterized by the decisive mathematical – i.e. geometrical – concepts and the corresponding explicit formulas:

1. Generalizing the basic concept of isospin (introduced originally by Heisenberg) we attach by now an n -dimensional (instead of a 2-dimensional) complex linear space to each point $(x^\mu, \mu=0, 1, 2, 3, \text{abbreviated by } x)$, of the basic 4-dimensional Minkowski-space (more general the Riemannian manifold) which supports the motion of elementary particles (e.g. electron, myon, quark) which are to be considered. In mathematical language such structures are called “fibre bundles”: They are the basic concept of present-day topology and represent the quantum-theoretical phase-space of a particle with “inner degree of freedom” in an appropriate and most intuitive way. The (complex) “vectors” of these fibres thus represent (according to the physical choice) e.g. spin, isospin, flavour and – most important for our purposes – colour (with $n=3$), i.e. the characteristic property of the quarks.

2. The decisive new concept of Gauge-theory is a (N -dimens.) Lie-group, in this connection the so-called Gauge-group (in most cases just $SU(n)$), which acts on (i.e. “rotates”) the vectors within these fibres. In contrast to the classical case of conventional spin, these (“symmetry”) groups generally appear completely disconnected from (coordinate-)transformations on the base-space (e.g. the Minkowski-space).

3. After these (geometrically most natural) preliminaries the decisive *local Gauge-principle* may be formulated as follows:

The basic (physical) laws have to be invariant under the general (i.e. space-time dependent or local) action

of the Gauge-group, i.e. the “rotations” within the various fibres which are attached to each point x are supposed to be independent of one another.

In contrast to the conventional (i.e. space-time independent or global) case, this enormously enlarged local invariance-principle thus exhibits an arbitrary space-time dependence of the Group-action. (This is in perfect analogy to Einstein's theory of Gravitation which in a certain sense represents a special example in which the tangent space represents our fibre bundle. Einstein's theory contains, however, some additional constraints due to the existence of the underlying metric tensor $g_{\mu\nu}$.)

4. The physical consequences of this (for a physical law most restrictive, or better, maximal) invariance principle (corresponding to Heisenberg's quest for symmetry [1]) are, in fact, decisive:

(a) It enforces a “field-action” between the (elementary) fermions (endowed, in turn, with a typical inner degree of freedom). The best known example – apart from the gravitational case – is the Maxwellian field, whereby (according to H. Weyl 1929) the characteristic fibre-bundle is given by the 1-dimensional complex plane, exhibiting the Rotation-group $U(1)$ which constitutes the basic property of wave-functions for *charged* particles. In both cases a former “action at a distance” – i.e. Newton and Coulomb – appears replaced by a basic field action!

(b) The explicit *implementation* of the local Gauge-principle, which leads to enormous physical consequences, is carried through by a mathematically (i.e. geometrically) well-known procedure, which constitutes, at the same time, an essential argument in Einstein's theory:

The (enlarged) infinitesimal “parallel-transport” (in geometry we speak about a connection-form) of the vectors v within the characteristic fibres is given by the action of an infinitesimal element of the symmetry-group, which in turn is a linear combination of the elements τ_a ($a=1, \dots, N$) of the corresponding Lie-algebra. The coefficients A^a depend on the direction of the displacements δx^μ and (this is the decisive property!) on the (local) position x within the Minkowski-space. Our parallel transport thus reads (using the “summation rule”)

$$\delta v(x) = \delta x^\mu A_\mu^a(x) \tau_a v(x), \quad (1)$$

whereby the Lie-elements τ_a are represented by $(n \otimes n)$ -matrices acting linearly on the n -dimensional

vectors v . The space-time, i.e. x -dependent coefficients $A_\mu^a(x)$ (i.e. a covariant vector field) now represent (“physically”) the “basic” Gauge-field, which is to be determined by gauge-invariant field equations. For this purpose (following the general procedure) one has to form the covariant derivation D_μ :

$$D_\mu \stackrel{\text{def}}{=} \partial_\mu + g A_\mu^a(x) \tau_a. \quad (2)$$

(g represents the “coupling-constant” introduced for practical purposes.) D_μ replaces (as usual) the “ordinary” derivation ∂_μ within the Dirac-wave equation of the basic fermions, which reads (in abbreviated spin-notation):

$$\left(\gamma^\mu D_\mu + i \frac{mc}{\hbar} \right) \psi(x) = 0. \quad (3)$$

The next and decisive step (basic in Riemannian Geometry and therefore also in General Relativity) represents the construction of the “Curvature-tensor” or “Field-strength” $F_{\mu\nu}^a(x)$ with the help of the famous commutator

$$F_{\mu\nu}^a \tau_a = [D_\mu, D_\nu]_-, \quad (4)$$

which (in contrast to the A 's) transforms gauge-covariantly and leads directly to the totally invariant (i.e. with respect to the *local* Gauge-transformations and the Lorentz-group) Lagrangian \mathcal{L} (omitting the usual matterpart):

$$\mathcal{L} = F_{\mu\nu}^a F_a^{\mu\nu}. \quad (5)$$

The corresponding Variation-principle (for A) then leads to the basic (locally invariant!) free field equations. Their typical *non*-linearity in the case of general (i.e. *non*-Abelian) Gauge-groups immediately follows from (4): The non-commutativity of the Lie-elements τ_a yields the “additional” non-linear term $\sim g_{bc}^a A_\mu^b A_\nu^c$ (containing the structure constants g_{bc}^a of the gauge-group) within the expression (4) of F .

5. This most natural and fundamental geometric scheme represents, in fact, the formal structure (by using different Gauge-groups) of our 3 basic physical laws, i.e. Gravitation (group $SL(4)$), Electroweak (group $SU(2) \otimes U(1)$) and Strong interaction (group $SU(3)$). Whereas the first two contain additional quantities (i.e. the metric field $g_{\mu\nu}$ and the Higgs-term, respectively) which hamper a satisfactory quantisation, the 3rd one represents a pure (and therefore renormalisable!) Gauge-theory, i.e. QCD based on 3-valued colour and the corresponding Gauge-group $SU(3)$.

It represents the underlying theory of the nearly unlimited realm of Hadrons, including, of course, Nuclear structures [5]: All phenomena of this domain are by now to be visualized as processes within systems of quarks bound exclusively by the action of the quantized Gauge-field, or, as expressed in quantum-theoretical terms, by gluon-exchange (in an enlarged sense, i.e. Instanton-exchange). Thus, a most intuitive but far reaching symmetry (according Heisenberg a concept which appears to be more fundamental than particles) represents the theoretical basis for the largest body of empirical data there is in experimental physics. (A similar situation holds with respect to electrons and the *commutative* Gauge-group $U(1)$, i.e. QED for Atoms and Molecules.) Realizing that there is not the slightest indication for a possible failure, the explicit (i.e., numerical) application of QCD represents, however, (in view of the characteristic non-linearity and the large coupling strength, this theory calls also for *non*-perturbative methods) an enormous mathematical and computational effort, or in other words, a great challenge for a whole generation of scientists.

A typical and fundamental “new” feature of QCD is the fact that the Hadrons – once considered to be elementary although their number is practically infinite – are by now (in perfect realisation of Heisenberg's quest!) systems of bound states (3 valence quarks, “sea”-quarks and an essential Gluonic part) based on one and the same basic interaction which yields – at the same time – an interpretation of these most striking (empirical) properties of their inner structure. Of course, these facts are intimately related to the “famous” confinement property of the quarks (closely related to the “non-linearity” of QCD), which, in a way, also exemplifies Heisenberg's ideas that the “new” basic fields (by now the spinorial quark field) might *not* have the usual “naive” physical interpretation, because individual quarks are, as a consequence of confinement, *not* directly observable particles.

6. We have thus realized that Heisenberg's general suggestions are fulfilled in an enlarged sense:

Particles, interpreted as bound states, are based on a new but rather abstract quantized field, exhibiting an essential non-linearity. It appears, however, unfortunate that the major input to present-day theory, namely the existence of quarks with their colour-degree of freedom, was not available at that time. The same holds for the fact that – apart from a few special

cases – *only* local Gauge-theories are renormalizable, i.e. lead at least to well defined numerical results. Heisenberg's model, in which the (lacking) field theoretical interaction between the elementary constituents appears replaced by a contact term (δ -function), represents (in analogy to the former, unsatisfactory Fermi-theory of Beta-decay) a *non*-renormalizable theory, a fact which led to a long series of “desperate” attempts trying to “cure” the “uncurable”. This rather extended work, which might – by now – be visualized as a special approximation scheme (e.g. the so-called nuclear “bag-model”) for present-day QCD, led, on the other side, to far-reaching mathematical insights into non-perturbative methods [2].

7. This general and most impressive fact that *all* fundamental laws in physics are governed by the same principle, i.e. the local Gauge-principle, suggested again (after Einstein's and Heisenberg's unsuccessful attempts) the idea of an all-embracing fundamental theory to be – tentatively – realized within the framework of so-called String-theory. There, geometry (in broadest sense) once more replaces the particle concept with the help of a perfectly new, but abstract, field theory. In fact, the so-far remaining elementary particles (Quarks as well as the leptons uncluding the Gravitons!) were to be interpreted as the field-quanta of a characteristic 2-dimensional basic field, or tube of string shape, embedded into a high-dimensional abstract space. As an attractive feature, the interaction

between these particles appears to be represented by a (topologically non-trivial) “soldering” of these tubes. In view of the (extreme) weakness of the gravitational interaction the diameter of the tubes had to be, however, of the order of the Planck-length, i.e. a region in which – historically for the first time – the validity of conventional (i.e. Archimedian) space-time structure was “seriously” questioned, or in other words, where the use of the conventional geometry might be (according Pauli!) the cause of the characteristic difficulties appearing all-over (microscopic) physical theories. For such reasons, Connes, Kastler, and many others were led to introduce (for this small length scale) a far-reaching new and basic geometry, i.e. the so-called “non-commutative differential geometry”. The need of a basic change – i.e. not only in large dimensions – was already suggested in general terms by Riemann in his “famous” thesis. This new scheme [7] corresponds – in a way – to a “quantisation of space”, in as much as the commutative ring of conventional functions, which defines geometrically the (classical) manifold (space), appears replaced by a *non*-commutative structure. This attempt (far from being fully realized, but useful also for practical purposes) might, in a way, meet Pauli's quest mentioned above. On the other side, a perfectly new and basic chapter of research appears to be an extreme consequence of Heisenberg's idea of “geometrisation” of particle physics.

- [1] Werner Heisenberg, *Der Teil und das Ganze*, Piper, Munich 1939, cf. p. 224 (above) where Heisenberg surveys his general principles; in particular he states – foreshadowing recent ideas in modern mathematical developments: According to Plato, symmetries appear to be more fundamental than particles!
- [2] W. Heisenberg, *Z. Naturforsch.* **5a**, 251, 367 (1950); **6a**, 281 (1951); **9a**, 292 (1954). – W. Heisenberg, F. Kortel, and H. Mitter, *Z. Naturforsch.* **10a**, 425 (1955). – R. Ascoli and W. Heisenberg, *ibid.* **12a**, 177 (1957). – H.-P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder, and K. Yamazaki, *ibid.* **14a**, 441 (1959). – H.-P. Dürr and W. Heisenberg, *ibid.* **16a**, 726 (1961). – W. Heisenberg, *Einführung in die einheitliche Feldtheorie der Elementarteilchen*, S. Hirzel-Verlag, Stuttgart 1967.
- [3] Texts on QCD are found a.o. in: P. Becher et al., *Eichtheorien*, Teubner, Stuttgart 1983; K. Huang, *Quarks, Leptons...*, World Scientific 1982; F. J. Yndurain, *Quantum Chromodynamics*, World Scientific 1983.
- [4] M. B. Green et al., *Superstring Theory*, Cambridge University Press, Cambridge 1986, and a large number of further publications.
- [5] K. Bleuler, M. Werner, *Pairing Theory and Diquarks in Workshop on Diquarks* (M. Anselmo, ed.), World Scientific 1989, p. 98. – K. Bleuler, *QCD and Nuclear Structure*, in: *Symmetries in Science II* (B. Gruber, ed.), Plenum Publ. 1988. – H. R. Petry, H. Hofestädt, S. Merk, K. Bleuler, H. Bohr, N. S. Narain, *Phys. Lett.* **159B**, 363.
- [6] Compare e.g. J. Koller and P. van Baal, *Nuclear Physics B* **302**, 1 (1988).
- [7] cf. e.g. E. Witten, *Nuclear Physics B* **268**, 255 (1989). – A. Connes, *Non-Commutative Diff. Geom.*, IHES preprints 1981 and 1983. – Yu. I. Manin, *Quantum Groups and Non-Can. Geom.*, publ. by Univ. of Montreal, CRM-1561 (July 1988). – D. Kastler, *Com. Math. Phys.* **121**, 345 (1989).
- [8] Compare e.g.: *Differential geom. Methods in theoret. Physics* (16th meeting held at Como) (K. Bleuler and M. Werner, eds.), Kluwer Acad. Publ. 1988, NATO-Series C, Vol. 250.
In addition: With same title (19th meeting at Rapallo). Report (U. Bruzzo et al., eds), to be published 1991 by Springer Lecture-Notes in Physics.